Oceanic Turbulence: As seen by GLAD

CARTHE Workshop

Coconut Grove, Florida

May 29-31, 2013

"I shall not today attempt to further define {...}.
... but I know it when I see it." (Potter Stewart, 1964)

Tennekes & Lumley:

- Irregular (random?) motion.
- Diffusive - rapid mixing of mass & momentum.
- Continuum - (know equations).
- Large Reynolds Number - multiple scales.
- Dissipative - energy lost.
- 3D vorticity fluctuations.

"Turbulence is the norm, not the exception.", JLL

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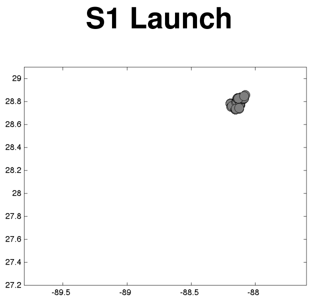

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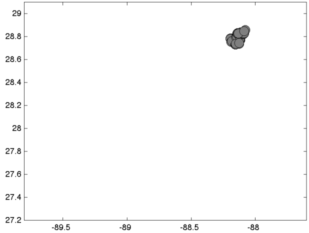

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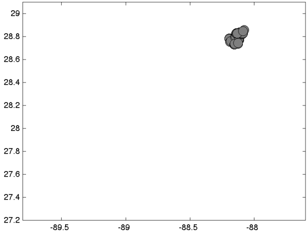

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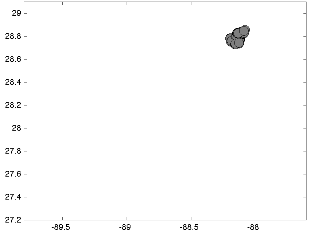

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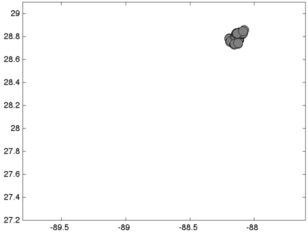

"In nature, flows can obtain Re $> 10^7$ *and scale separation can be very large."*

Goals: How turbulent was GLAD?

Monin & Yaglom view of GLAD data:

- Lagrangian observations⇔ Eulerian velocity field
  - Scale dependence of velocity fluctuations
  - Wavenumber spectrum

- Assume *Turbulence*:
  - Theory ⇔ Data (?)
  - Which *turbulence*? (2D-3D?)

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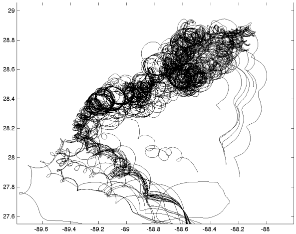

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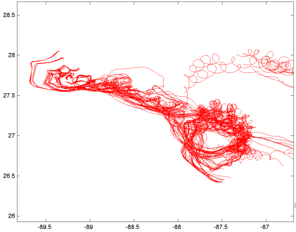

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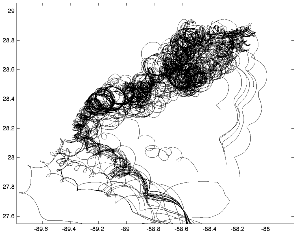
- Mesoscale-Submesoscale Boundary
  - Timescales:
    minutes $\leq \tau \leq$ weeks .
  - Lengthscales:
    100 meters $\leq r \leq$ 100 kilometers.

S1 Launch



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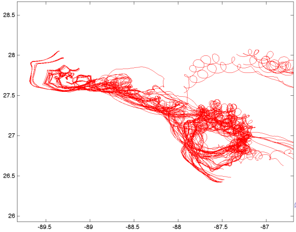
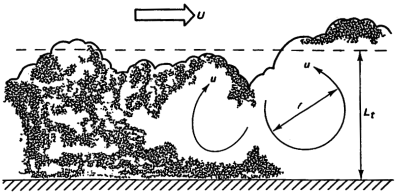


100+ years of Turbulence Theory (in ∼10 minutes)



Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.
– Lewis F. Richardson,1920

Assumptions:

- $\mathrm{Re} = \frac{UL_t}{\nu} \gg 1$
- Energy:
  - Input at large-scales: L_t.
  - Viscous dissipation at scale η where $\frac{\eta u}{\nu} \sim 1$
- Cascade of energy from L_t to η.
- In *inertial range*: $\eta \ll l \ll L_t$
  - Statistics independent of both:
    - ★ Specifics of large scale forcing.
    - ★ Specifics of small scale dissipation. (ν itself).
  - Only system parameter is Cascade Rate:
    $\varepsilon =$ Energy dissipation rate.

Two-Point Statistics:

- Correlations:

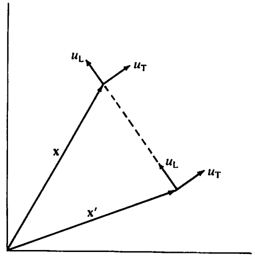
$$B_{ij}(\mathbf{x}, \mathbf{x}', t, t') = \langle u_i(\mathbf{x}, t) u_j(\mathbf{x}', t') \rangle$$

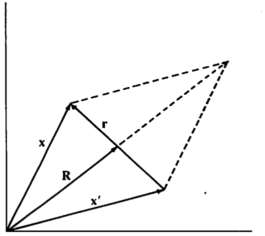
- Stationary and Homogeneous:

$$B_{ij}(\mathbf{R}, \mathbf{r}, t, t') = B_{ij}(\mathbf{r}, t - t')$$

- Isotropic:

$$B_{ij}(\mathbf{r}, t - t') = B_{ij}(\|\mathbf{r}\|, t - t') = B_{ij}(r, t - t') =$$





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- Isotropic:

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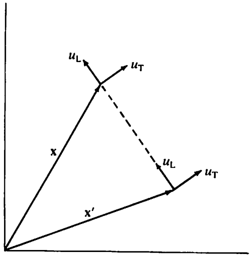
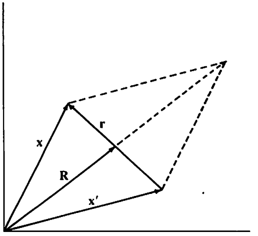
- New Coordinates:

$$B_{ll}(r, \tau) = \langle u_l(r, t + \tau) u_l(0, t) \rangle$$

$$B_{\perp\perp}(r, \tau) = \langle u_T(r, t + \tau) u_T(0, t) \rangle$$





One-time, Two-Point Correlations - Energy spectra

- Stationary, Homogeneous:

$$R_{ij}(\mathbf{r}) = \langle u_i(\mathbf{x}+\mathbf{r},t)u_j(\mathbf{x},t)\rangle$$

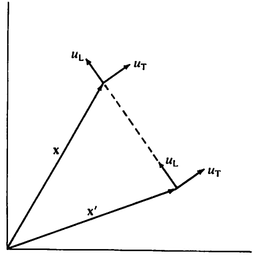
- Fourier Transform:

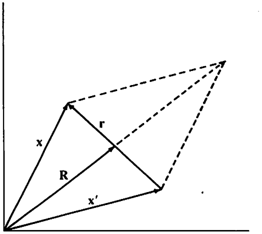
$$R_{ij}(\mathbf{r}) = \mathcal{F}^{-1}\left\{\phi_{ij}(\mathbf{k})\right\}$$

- Spectral Energy Density:

$$R_{ii}(0) = \langle u_i(\mathbf{x},t)u_i(\mathbf{x},t)\rangle = \iiint_{-\infty}^{\infty} \phi_{ii}(\mathbf{k})d\mathbf{k}$$

$$\mathcal{E}(k) = \frac{1}{2}\oiint \phi_{ii}(\mathbf{k})d\sigma, \quad E = \int_0^{\infty} \mathcal{E}(k)dk$$





Local Isotropy - Structure Functions:

- Velocity increment:

$$\Delta_{\mathbf{r}}\mathbf{u} = \mathbf{u}(\mathbf{x}+\mathbf{r},t) - \mathbf{u}(\mathbf{x},t)$$

- In *inertial* range: $P(\Delta_r \mathbf{u})$ stationary, homogeneous and isotropic.
- Structure functions:

$$S_p^l(r) = \langle (\Delta_r u_L)^p \rangle$$

- In *inertial* range: $S_p(r) = f(\varepsilon, r)$

- By dimensional arguments:

$$S_2^l(r) = \langle (\Delta_r u_L)^2 \rangle \approx C\varepsilon^{2/3} r^{2/3}$$

- In wavenumber space:

$$\mathcal{E}(k) = C'\varepsilon^{2/3} k^{-5/3}$$

- Define local timescale:

$$\tau(r) = r\left(\langle (\Delta_r u_L)^2 \rangle\right)^{-1/2}$$

$$\tau(r) \approx r^{2/3}$$

Local, 3D Energy Cascade:
$\mathcal{E}(k) = g(k, \varepsilon), \ [\varepsilon] = L^2/T^3$

- Only dimensionally consistent relation:

$$\mathcal{E}(k) = \mathcal{K}\varepsilon^{2/3}k^{-5/3}$$

- Physical space:

$$S_2(r) \sim r^{2/3}$$

- Local time-scale:

$$\tau_r \sim \frac{r}{\sqrt{S_2(r)}} \sim r^{2/3}$$

Local, 2D Enstrophy Cascade:
$\mathcal{E}(k) = g(k, \eta), \ [\eta] = 1/T^3$

- Only dimensionally consistent relation:

$$\mathcal{E}(k) = \mathcal{K}_\eta \eta^{2/3}k^{-3}$$

- Physical space:

$$S_2(r) \sim r^2$$

- Local time-scale:

$$\tau_r \sim \frac{r}{\sqrt{S_2(r)}} \sim \mathrm{Const}$$

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Richardson '26:

$$D^2(t) \sim t^3$$

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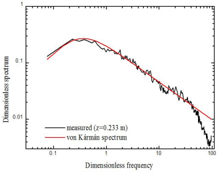
Exponential:

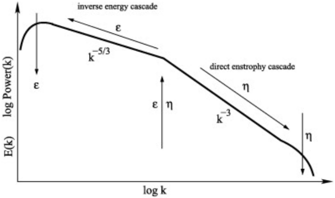
$$\lim_{\delta \to 0} \lambda(\delta) = \lambda_0$$

3D Kolmogorov & 2D Kraichnan:

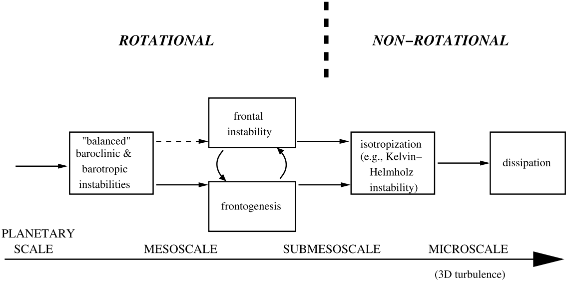
3D Turbulence: Wittwer *et al*



2D Turbulence: Vallis

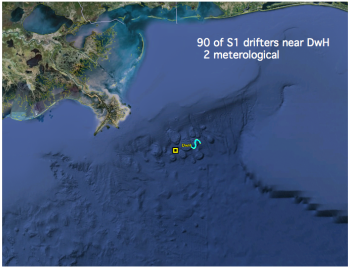


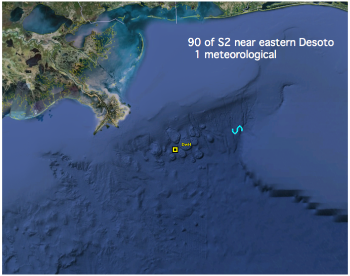
Major Question: How does geostrophic ocean dissipate energy?

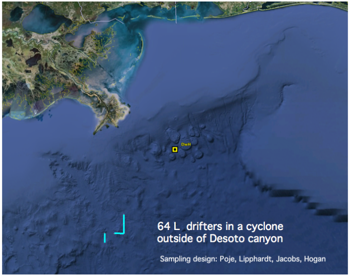


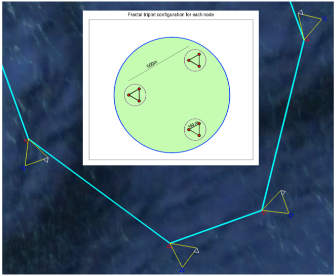
(McWilliams, 2010)

GLAD: Observing Relative Dispersion at ~Submesoscales



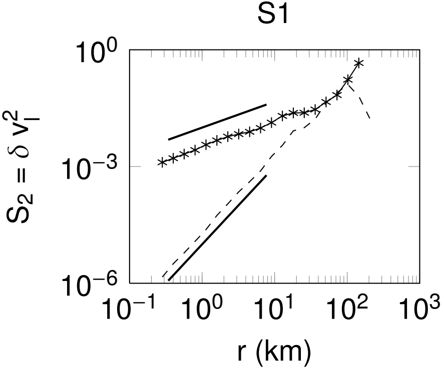


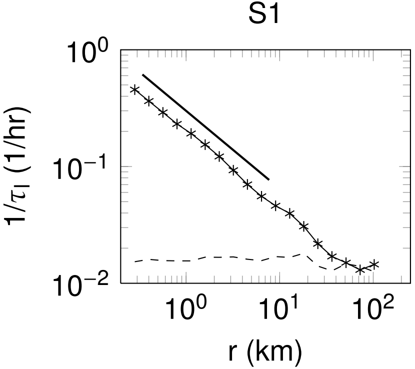




Comparisons:

- In Canyon: S1 and S2
  - Strong frontal signatures in T-S.
  - Strong diurnal/inertial signals.
  - Slow spreading (S1), high data density at small scales.
- 'Open Ocean': C1
  - Targeted energetic cyclonic eddy ($l \sim \mathcal{O}(30km)$).
  - Diurnal/inertial signal less dominant.
  - Lower data density at small scales.
- Altimetry Data: (S1 and S2)
  - *Olascoaga, Beron-Vera, Iskandarani*.
  - Strictly geostrophic velocities from observations.



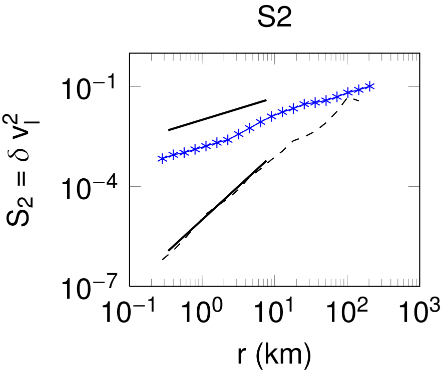


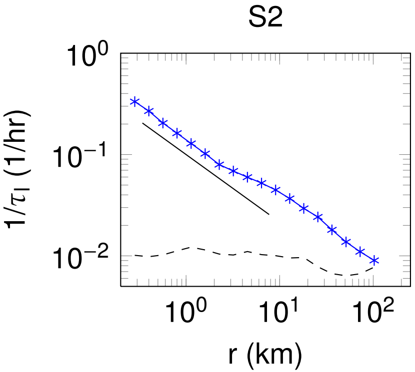
$S_2(r) \sim r^{2/3}$

$$\frac{1}{\tau_r} \sim r^{-2/3}$$

$S_2(r) \sim r^2$

$$\frac{1}{\tau_r} \sim \mathrm{Const}$$





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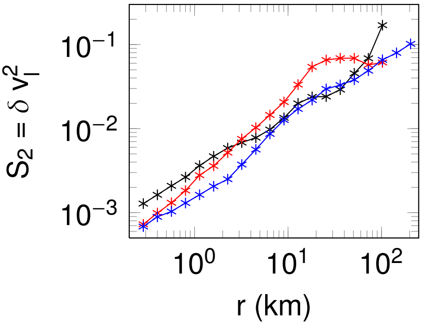
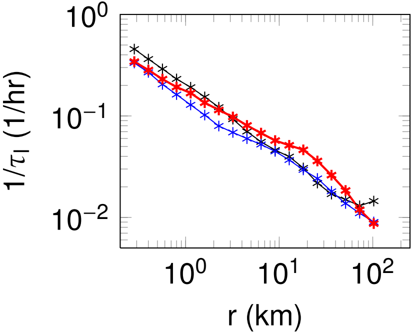




$S_2 \sim r^{\beta}$

$\mathcal{E}(k) \sim k^{-(1+\beta)}$

- S1: $\beta = 0.66$
- S2: $\beta = 0.67$
- C1: $\beta = 1.08$
- Altimetry: $\beta = 2.0$

- S1: $\mathcal{E}(k) \sim k^{-5/3}$
- S2: $\mathcal{E}(k) \sim k^{-5/3}$
- C1: $\mathcal{E}(k) \sim k^{-2}$
- Altimetry: $\mathcal{E}(k) \sim k^{-\gamma}$, $\gamma \geq 3$

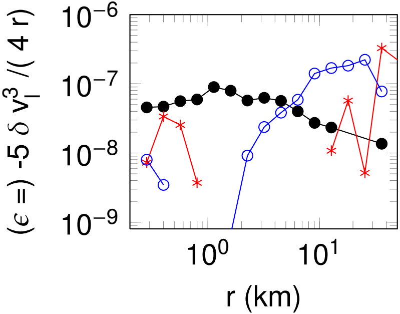
Grain of salt: $\mathcal{E}(k) \sim k^{-5/3}$ does not imply 3D turbulence.

- 4/5 Law: 'Exact' Relation:

(strong isotropy)

$$\langle(\delta v_l^3)\rangle = -\frac{4}{5}\varepsilon r$$

  - S1 $\Longrightarrow$ energy cascade(?)
  - Value of ε comparable to microstructure?



- Lagrangian structure functions:

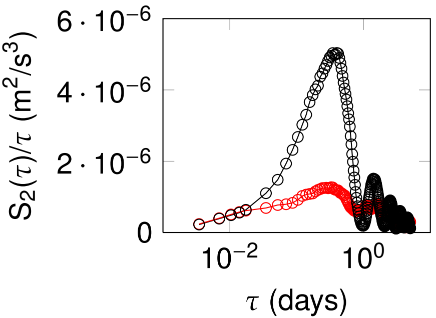
$$S_p(\tau) = \langle (v_l(t+\tau) - v_l(t))^p \rangle$$

  - Inertial range: $\tau_k \ll \tau \ll T$:

$$S_p(\tau) = g(\varepsilon, \tau)$$

$$S_2(\tau) \sim \varepsilon\tau$$

  - *Dissipation* in S1 > C1.
  - *Inertial* range $\tau \sim \mathcal{O}$(hours).



GLAD: Difference between S1 and Cyclone

- Absolute Dispersion: Taylor 1921

$$y(t,a) = x(t,a) - a = \int_0^t v(t')dt'$$

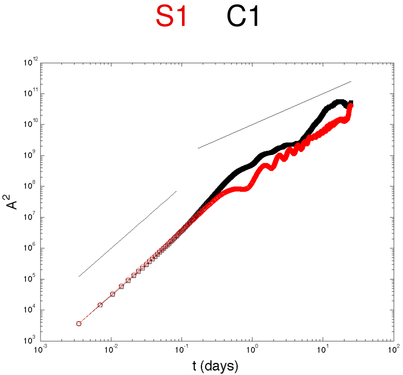
- Lagrangian velocity correlation:

$$\langle v^2 \rangle R(\tau) = \langle v(\tau) v(0) \rangle$$

$$\langle y^2(t) \rangle = 2\langle v^2 \rangle \int_0^t (t-\tau) R(\tau) d\tau$$

$$\mathcal{T}_L = \int_0^\infty R(\tau) d\tau$$

$$\langle y^2(t) \rangle = \begin{cases} \langle v^2 \rangle t^2 & t \ll \mathcal{T}_L \\ \langle 2v^2 \rangle t \mathcal{T}_L & t > \mathcal{T}_L \end{cases}$$



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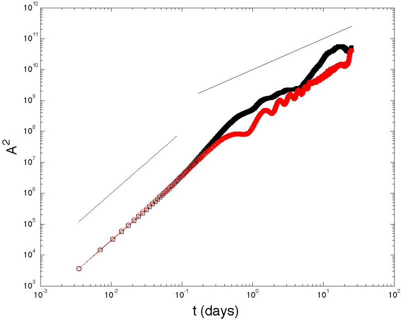
S1 C1



$$\mathcal{T}_L \approx \mathcal{T}_L$$

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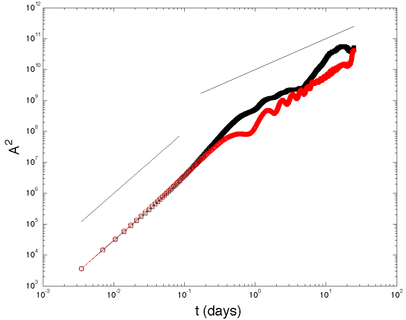

$$\mathcal{T}_L \approx \mathcal{T}_L$$

$$\langle v^2 \rangle < \langle v^2 \rangle$$

- Similar absolute dispersion.
- Distinctly different relative dispersion & spectra.
- Different forcing?
- Frequency spectra of

$$S_2(\tau) = \langle (v_l(t+\tau) - v_l(t))^2 \rangle$$

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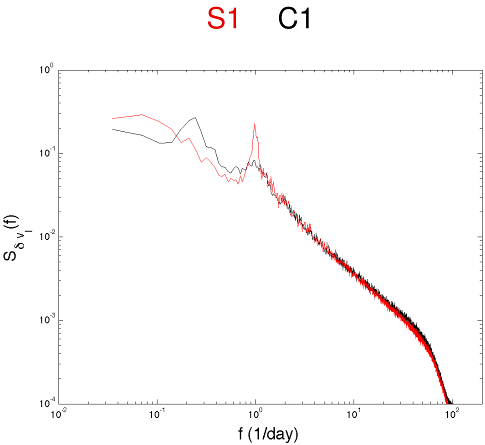

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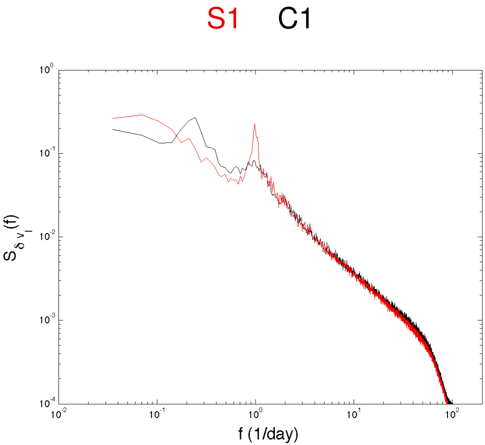
- Strong inertial signal in difference spectrum.



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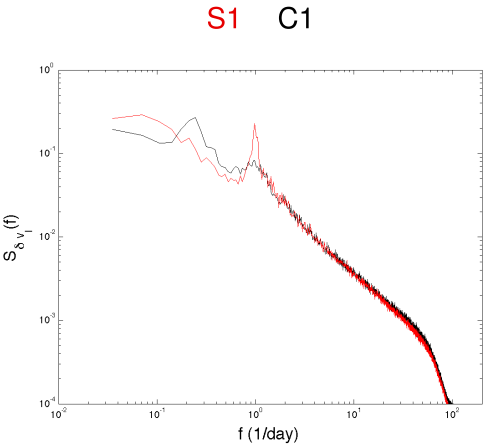
- Strong inertial signal in difference spectrum.
- See Emanuel Coelho (tomorrow).



Summary: Turbulence at ocean surface from GLAD

- Canyon launches S1 (S2):

$$S_2(r) \sim r^{2/3}, \quad r \le (2-3)\mathrm{km}$$

  - Two point statistics (scale dependent relative dispersion, Eulerian & Lagrangian structure functions, timescales) entirely consistent with forward cascade of energy.
  - S1: $\sim$constant ε in *inertial range*: $r < 5km$.

- Cyclone:

$$S_2(r) \sim r^1, \quad r \le \sim 10\mathrm{km}$$

  - Two point statistics clearly inconsistent with steep ($\beta \ge 3$) spectra.
  - Not classical *2D*, geostrophic turbulence.

Local dispersion regime in all cases.

- Homogeneous & Isotropic?
  - Can be readily checked.
- Statistics?
  - Need error bars, badly.
  - Highly non-gaussian statistics.
- Physics? (Especially in S1/S2 launches.)
  - Energy input at inertial radius $\Rightarrow$ forward cascade through submesoscales (?)
  - Random inertial waves $\Rightarrow \mathcal{E}(k) \sim k^{-}5/3(?)$

- Homogeneous & Isotropic?
  - Can be readily checked.
  - Will do so, Monday.
- Statistics?
  - Need error bars, badly.
  - Highly non-gaussian statistics.
- Physics? (Especially in S1/S2 launches.)
  - Energy input at inertial radius $\Rightarrow$ forward cascade through submesoscales (?)
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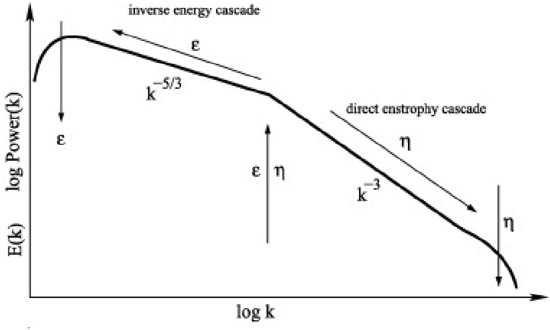

"In nature, flows can obtain Re $> 10^7$ *and scale separation can be very large."* ...