Oceanic Turbulence: As seen by GLAD

CARTHE Workshop

Coconut Grove, Florida

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Turbulence?

“I shall not today attempt to further define {...}. … but I know it when I see it.” (Potter Stewart, 1964)

Tennekes & Lumley:

- Irregular (random?) motion.
- Diffusive - rapid mixing of mass & momentum.
- Continuum - (know equations).
- Large Reynolds Number - multiple scales.
- Dissipative - energy lost.
- 3D vorticity fluctuations.

“Turbulence is the norm, not the exception.”, JLL
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“In nature, flows can obtain $Re > 10^7$ and scale separation can be very large.”
Goals: How turbulent was GLAD?

Monin & Yaglom view of GLAD data:

- Lagrangian observations ⇔ Eulerian velocity field
  - Scale dependence of velocity fluctuations
  - Wavenumber spectrum

- Assume Turbulence:
  - Theory ⇔ Data (?)
  - Which turbulence? (2D-3D?)
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- Mesoscale-Submesoscale Boundary
  - Timescales: minutes ≤ \( \tau \) ≤ weeks.
  - Lengthscales: 100 meters ≤ \( r \) ≤ 100 kilometers.
Big whorls have little whorls 
That feed on their velocity, 
And little whorls have lesser 
whorls 
And so on to viscosity.  
– Lewis F. Richardson, 1920

Assumptions:

- \( \text{Re} = \frac{UL_t}{\nu} \gg 1 \)
- Energy:
  - Input at large-scales: \( L_t \).
  - Viscous dissipation at scale \( \eta \) where \( \frac{\eta u}{\nu} \sim 1 \)
- Cascade of energy from \( L_t \) to \( \eta \).
- In inertial range: \( \eta \ll l \ll L_t \)
  - Statistics independent of both:
    - Specifics of large scale forcing.
    - Specifics of small scale dissipation. (\( \nu \) itself).
  - Only system parameter is Cascade Rate:
    \( \varepsilon = \text{Energy dissipation rate} \).
Two-Point Statistics:

- **Correlations:**
  \[ B_{ij}(\mathbf{x}, \mathbf{x}', t, t') = \langle u_i(\mathbf{x}, t)u_j(\mathbf{x}', t') \rangle \]

- **Stationary and Homogeneous:**
  \[ B_{ij}(\mathbf{R}, \mathbf{r}, t, t') = B_{ij}(\mathbf{r}, t - t') \]

- **Isotropic:**
  \[ B_{ij}(\mathbf{r}, t-t') = B_{ij}(\|\mathbf{r}\|, t-t') = B_{ij}(\mathbf{r}, t-t') = \]
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- **New Coordinates:**
  \[ B_{\parallel}(r, \tau) = \langle u_i(r, t + \tau)u_i(0, t) \rangle \]
  \[ B_{\perp\perp}(r, \tau) = \langle u_T(r, t + \tau)u_T(0, t) \rangle \]
One-time, Two-Point Correlations - Energy spectra

- **Stationary, Homogeneous:**

\[ R_{ij}(r) = \langle u_i(x + r, t) u_j(x, t) \rangle \]

- **Fourier Transform:**

\[ R_{ij}(r) = \mathcal{F}^{-1} \{ \phi_{ij}(k) \} \]

- **Spectral Energy Density:**

\[ R_{ii}(0) = \langle u_i(x, t) u_i(x, t) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{ii}(k) \, dk \]

\[ \mathcal{E}(k) = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{ii}(k) \, d\sigma, \quad E = \int_{0}^{\infty} \mathcal{E}(k) \, dk \]
Local Isotropy - Structure Functions:

- **Velocity increment:**
  \[ \Delta_r u = u(x + r, t) - u(x, t) \]

- In *inertial* range: \( P(\Delta_r u) \) stationary, homogeneous and isotropic.

- **Structure functions:**
  \[ S_p^l(r) = \langle (\Delta_r u_L)^p \rangle \]

- In *inertial* range: \( S_p(r) = f(\varepsilon, r) \)

- By dimensional arguments:
  \[ S_2^l(r) = \langle (\Delta_r u_L)^2 \rangle \approx C \varepsilon^{2/3} r^{2/3} \]

- In wavenumber space:
  \[ \mathcal{E}(k) = C' \varepsilon^{2/3} k^{-5/3} \]

- **Define local timescale:**
  \[ \tau(r) = r \left( \langle (\Delta_r u_L)^2 \rangle \right)^{-1/2} \]
  \[ \tau(r) \approx r^{2/3} \]
Local, 3D Energy Cascade: $E(k) = g(k, \varepsilon), \ [\varepsilon] = L^2/T^3$

- Only dimensionally consistent relation:
  \[ E(k) = K \varepsilon^{2/3} k^{-5/3} \]

- Physical space:
  \[ S_2(r) \sim r^{2/3} \]

- Local time-scale:
  \[ \tau_r \sim \frac{r}{\sqrt{S_2(r)}} \sim r^{2/3} \]

Local, 2D Enstrophy Cascade: $E(k) = g(k, \eta), \ [\eta] = 1/T^3$

- Only dimensionally consistent relation:
  \[ E(k) = K \eta^{2/3} k^{-3} \]

- Physical space:
  \[ S_2(r) \sim r^2 \]

- Local time-scale:
  \[ \tau_r \sim \frac{r}{\sqrt{S_2(r)}} \sim \text{Const} \]
3D Kolmogorov & 2D Kraichnan:

Local, 3D Energy Cascade:
\[ E(k) = g(k, \varepsilon), \quad [\varepsilon] = L^2 / T^3 \]

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Richardson '26:
\[ D^2(t) \sim t^3 \]

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Exponential:
\[ \lim_{\delta \to 0} \lambda(\delta) = \lambda_0 \]
3D Kolmogorov & 2D Kraichnan:

3D Turbulence: Wittwer et al

2D Turbulence: Vallis

Major Question: How does geostrophic ocean dissipate energy?

A Perspective on Submesoscale Geophysical Turbulence

James C. McWilliams

1 The Dynamical Regime of Submesoscale Turbulence

Define the submesoscale regime as the geophysical fluid dynamics that arise from advective processes and that have a marginal degree of dynamical control by planetary rotation and stable density stratification. The degrees are conventionally measured by a Rossby number, \[ R_o = \frac{V}{fL} \] (where \( V \) is a horizontal speed, \( f \) is the Coriolis frequency, and \( L \) is a horizontal length), and a Froude number, \[ F_r = \frac{V}{NH} \] (where \( N \) is a stratification constant).

Fig. 1

Sketch of the flow of dynamical control and energy from the global forcing of the general circulation through the balanced mesoscale and partially-balanced submesoscale ranges down to the isotropic microscale where dissipation occurs.

James C. McWilliams

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(McWilliams, 2010)
GLAD: Observing Relative Dispersion at ~Submesoscales
GLAD: Submesoscale Relative Dispersion

Comparisons:

- In Canyon: S1 and S2
  - Strong frontal signatures in T-S.
  - Strong diurnal/inertial signals.
  - Slow spreading (S1), high data density at small scales.

- ‘Open Ocean’: C1
  - Targeted energetic cyclonic eddy ($l \sim O(30\text{km})$).
  - Diurnal/inertial signal less dominant.
  - Lower data density at small scales.

- Altimetry Data: (S1 and S2)
  - *Olascoaga, Beron-Vera, Iskandarani*.
  - Strictly geostrophic velocities from observations.
$S_2 = \delta v^2_l$

$S_2(r) \sim r^{2/3}$

$S_2(r) \sim r^2$

$\frac{1}{\tau_r} \sim r^{-2/3}$

$\frac{1}{\tau_r} \sim \text{Const}$
GLAD: Submesoscale Relative Dispersion

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\[ \frac{1}{\tau_r} \sim \text{Const} \]
GLAD: Submesoscale Relative Dispersion

\[ S_2 = \delta v_1^2 \]

\[ S_2 \sim r^\beta \]

\[ \mathcal{E}(k) \sim k^{-(1+\beta)} \]

- S1: \( \beta = 0.66 \)
- S2: \( \beta = 0.67 \)
- C1: \( \beta = 1.08 \)
- Altimetry: \( \beta = 2.0 \)

- S1: \( \mathcal{E}(k) \sim k^{-5/3} \)
- S2: \( \mathcal{E}(k) \sim k^{-5/3} \)
- C1: \( \mathcal{E}(k) \sim k^{-2} \)
- Altimetry: \( \mathcal{E}(k) \sim k^{-\gamma}, \gamma \geq 3 \)
GLAD: Kolmogorov’s 4/5 Law and Lagragian Structure functions:

Grain of salt: $\mathcal{E}(k) \sim k^{-5/3}$ does not imply 3D turbulence.

- **4/5 Law: 'Exact' Relation:**
  
  \[
  \langle (\delta v_i^3) \rangle = -\frac{4}{5} \mathcal{E} r
  \]

  - S1 $\implies$ energy cascade(?)
  - Value of $\mathcal{E}$ comparable to microstructure?
GLAD: Kolmogorov’s 4/5 Law and Lagrangian Structure functions:

- Lagrangian structure functions:

\[ S_p(\tau) = \langle (v_l(t + \tau) - v_l(t))^p \rangle \]

- Inertial range: \( \tau_k \ll \tau \ll T \):

\[ S_p(\tau) = g(\epsilon, \tau) \]

\[ S_2(\tau) \sim \epsilon \tau \]

- Dissipation in S1 > C1.
- Inertial range \( \tau \sim O(\text{hours}) \).
GLAD: Difference between S1 and Cyclone

- Absolute Dispersion: Taylor 1921

\[ y(t, a) = x(t, a) - a = \int_0^t v(t')dt' \]

- Lagrangian velocity correlation:

\[ \langle v^2 \rangle R(\tau) = \langle v(\tau)v(0) \rangle \]

\[ \langle y^2(t) \rangle = 2\langle v^2 \rangle \int_0^t (t-\tau)R(\tau)d\tau \]

\[ T_L = \int_0^\infty R(\tau)d\tau \]

\[ \langle y^2(t) \rangle = \begin{cases} \langle v^2 \rangle t^2 & t \ll T_L \\ \langle 2v^2 \rangle tT_L & t > T_L \end{cases} \]
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  \[ T_L \approx T_L \]
  \[ \langle v^2 \rangle < \langle v^2 \rangle \]
GLAD: Difference between S1 and Cyclone

- Similar absolute dispersion.
- Distinctly different relative dispersion & spectra.
- Different forcing?
- Frequency spectra of

\[ S_2(\tau) = \langle (v(t+\tau) - v(t))^2 \rangle \]
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- Strong inertial signal in difference spectrum.
- See Emanuel Coelho (tomorrow).
Canyon launches S1 (S2):

\[ S_2(r) \sim r^{2/3}, \quad r \leq (2 - 3)\text{km} \]

- Two point statistics (scale dependent relative dispersion, Eulerian & Lagrangian structure functions, timescales) entirely consistent with forward cascade of energy.
- S1: \( \sim \) constant \( \varepsilon \) in inertial range: \( r < 5\text{km} \).

Cyclone:

\[ S_2(r) \sim r^1, \quad r \leq \sim 10\text{km} \]

- Two point statistics clearly inconsistent with steep (\( \beta \geq 3 \)) spectra.
- Not classical 2D, geostrophic turbulence.

Local dispersion regime in all cases.
Questions: Grains of salt

- Homogeneous & Isotropic?
  ▶ Can be readily checked.

- Statistics?
  ▶ Need error bars, badly.
  ▶ Highly non-gaussian statistics.

- Physics? (Especially in S1/S2 launches.)
  ▶ Energy input at inertial radius $\Rightarrow$ forward cascade through submesoscales (?)
  ▶ Random inertial waves $\Rightarrow \mathcal{E}(k) \sim k^{-5/3}$(?)
Questions: Grains of salt

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  ▶ Can be readily checked.
  ▶ Will do so, Monday.

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